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A scattering method for the equilibrium spin current in a ferromagnet junction

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Abstract
We extended McMillan's Green's function method to study the equilibrium spin current (ESC) in a ferromagnet/ferromagnet (FM/FM) tunnelling junction, in which the magnetic moments in both FM electrodes are not collinear. The single-electron Green's function of the junction system is directly constructed from the elements of the scattering matrix, which can be obtained by matching wavefunctions at boundaries. The ESC is found to be determined only by the Andreev-type reflection amplitudes as in the Josephson effect. The obtained expression of ESC is an exact result and at the strong barrier limit gives the same explanation for the origin of ESC as the linear response theory, that is, ESC comes from the exchange coupling between the magnetic moments of the two FM electrodes, $\mathbf{J} \sim \mathbf{h}_{\mathrm{l}} \times \mathbf{h}_{\mathrm{r}}$. In the weak barrier region, ESC cannot form spontaneously in a noncollinear FM/FM junction when there is no tunnelling barrier between the two FM electrodes.

1. Introduction

Spin related transport in magnetic hybrid systems has been studied intensively for the last two decades and considerable progress has been achieved since the discovery of the tunnelling magnetoresistance (TMR) effect [1] in ferromagnet (FM) tunnelling junctions. This effect originates from the electrical resistances of these junctions being dependent strongly on whether the moments of adjacent magnetic layers are parallel or antiparallel. The reason is that the tunnelling electrons are scattered more strongly in an antiparallel magnetic configuration than in a parallel configuration. As a result, tunnelling junctions with moments in adjacent magnetic layers aligned antiparallel have larger overall resistance than when the adjacent moments are aligned parallel, giving rise to TMR. The reverse effect of TMR is the spin transfer effect predicted independently by Slonczewski [2] and Berger [3] about a decade ago, in which a sufficiently large spin-polarized current injected from a normal metal (NM) into an FM layer can lead to magnetic moment reversal in the FM layer.

Spin transfer torque occurs in magnetic multilayers with noncollinear moments and is due to the nonconservation of spin current through the interface between NM and FM. Due to the presence of noncollinearity, the component of spin current transverse to the magnetization of the layers is not transmitted across the interfaces between NM and FM; in other words, the discontinuity of spin current at the interface is the origin of the spin transfer effect that can result in magnetization precession or reversal [4–8]. A large number of experiments have observed this spin transfer effect in magnetic multilayer structures [9–11], in which one of the FM layers is very thick and works as a polarizer of electric current with its fixed moment, and the moment of the thin FM layer may switch when a strong polarized electric current perpendicular to the layer plane flows through the layers.

As the two magnetization directions in the FM/FM tunnelling junction are misaligned, an equilibrium spin current (ESC) can flow in the junction without any bias [7]; this phenomenon is analogous to the Josephson effect that the superconductor macroscopic phase difference between the two sides of a junction drives a supercurrent through the junction. The dissipationless ESC will disappear if two moments in the adjacent FM layers are collinear, parallel or antiparallel. The existence of ESC $\mathbf{J}$ has been verified by many authors using the linear response theory [12–16], and explained as the result of the exchange coupling [17] between two magnetic moments $\mathbf{h}_\mathrm{r}$ and $\mathbf{h}_\mathrm{l}$, $\mathbf{J} \sim \mathbf{h}_\mathrm{l} \times \mathbf{h}_\mathrm{r}$. Thus the magnetization phase difference between the FM electrodes induces ESC in the FM junction as in the Josephson effect. In the case of a thin barrier between FMs or the strong coupling limit, the behaviour of ESC is still unknown in the literature and is worth studying.

In this paper, we study the ESC flowing in an FM/FM junction with two fixed noncollinear magnetic moments using a simple quantum mechanical approach. The present work is motivated by two factors. Firstly, dissipationless spin current in last several years has drawn considerable interest and also incurred much debate, such as the controversies in the spin Hall effect [18–21]. The study of ESC in FM/FM junctions may shed light on the spin current in spin–orbit coupled systems. Secondly, the well known result of ESC $\mathbf{J} \sim \mathbf{h}_\mathrm{l} \times \mathbf{h}_\mathrm{r}$ in FM/FM junctions in the literature was obtained by using the linear response theory [12–16] or heuristic derivation [22], thus an exact result of ESC is in order. By extending McMillan's Green's function method, which was originally employed to study the Josephson effect in a superconductor junction [23], we obtained an exact expression of ESC in the FM/FM junction at arbitrary tunnelling-barrier strength. ESC is determined only by the Andreev-type reflection amplitudes, [8, 24] which is similar to the Josephson effect [25, 26]. The exact ESC can reproduce the well known result stemming from the linear response theory [12–16] in the strong barrier case, whereas in the opposite case the ESC was found to vanish when there is no barrier between the two FM electrodes, which indicates the ESC should be a pure quantum mechanical effect.

This paper is organized in the following way. In section 2, a FM/FM junction model is given and the single-electron Green's function of the junction system is constructed from the scattering coefficients. An analytic expression of ESC in the FM/FM junction is obtained in section 3 and some discussions are presented. A conclusion is drawn in the last section.

2. Green's function

The FM/FM tunnelling junction is depicted schematically in figure 1 and the layer between the FM electrodes can be either an insulator barrier or a normal metal. A simple free electron model is adopted to describe the FM/FM tunnelling junction and the Hamiltonian reads

$$\mathcal{H} = \frac{-\hbar^2\nabla^2}{2m_\mathrm{e}} + V(x) - \theta(-x)\mathbf{h}_\mathrm{l}\cdot\boldsymbol{\sigma} - \theta(x-L)\mathbf{h}_\mathrm{r}\cdot\boldsymbol{\sigma}, \tag{1}$$

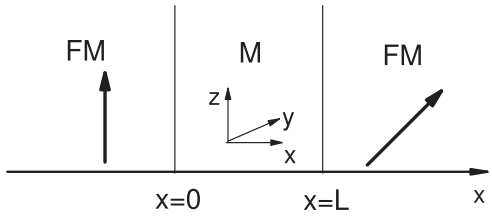

Figure 1. Schematic diagram of an FM/FM tunnelling junction with two noncollinear magnetizations in the left and right FM electrodes. The middle region M at $0 < x < L$ can be either an insulator barrier or a normal metal. The z-direction is taken as the spin quantization axis parallel to the left magnetization $\mathbf{h}_\mathrm{l}$.

where m_e is the effective electron mass and is assumed to be identical in all three regions, the two FM electrodes and the middle region. $\mathbf{h}_\mathrm{l}$ and $\mathbf{h}_\mathrm{r}$ are the internal molecular fields of the left and right FM, respectively. $\boldsymbol{\sigma}$ denotes the Pauli spin operator, and $\theta(x)$ is the step function. The potential energy $V(x)$ may take different values in different regions but remains constant in both FMs. Here the molecular fields $\mathbf{h}_\mathrm{l}$ and $\mathbf{h}_\mathrm{r}$ (magnetization with energy units) are not collinear in our consideration and are assumed to be fixed by an external magnetic field or other methods. Without loss of generality, we take the spin quantization axis of the system to be parallel to the magnetization of the left FM $\mathbf{h}_\mathrm{l}$ and the direction of $\mathbf{h}_\mathrm{r}$ of the right FM is described by the polar coordinate (θ, ϕ).

In a free electron model, the energy dispersions of the two FMs can readily be solved and they are spin dependent. In the left FM, the eigenvalue is $E_\pm = \frac{\hbar^2 k^2}{2m_\mathrm{e}} \pm h_\mathrm{l} + U_\mathrm{l}$ and the spinor is the eigenfunction of σ_z, and in the right FM, $E_\pm = \frac{\hbar^2 k^2}{2m_\mathrm{e}} \pm h_\mathrm{r} + U_\mathrm{r}$ and the spin eigenfunctions are

$$\psi_+^\mathrm{l} = \begin{pmatrix} \cos(\theta/2)\mathrm{e}^{-\mathrm{i}\phi/2} \\ \sin(\theta/2)\mathrm{e}^{\mathrm{i}\phi/2} \end{pmatrix} \qquad \text{and} \qquad \psi_-^\mathrm{l} = \begin{pmatrix} -\sin(\theta/2)\mathrm{e}^{-\mathrm{i}\phi/2} \\ \cos(\theta/2)\mathrm{e}^{\mathrm{i}\phi/2} \end{pmatrix}$$

where U_l (U_r) are the different potential energies on the left (right) FM, $\mathbf{k}$ is the wavevector of the electron and $\pm$ is the spin index. The spatial eigenfunctions are plane waves. Due to spin splitting, there are four incoming wavefunctions with their corresponding outgoing wavefunctions in the left and right FM, as schematically shown in figure 2. $\Psi_1(x)$ and $\Psi_2(x)$ are the wavefunctions of the minority spin and majority spin electrons injecting from the left FM, respectively, while $\Psi_3(x)$ and $\Psi_4(x)$ are those injecting from the right FM. For example, the wavefunction of the first type of scattering event $\Psi_1(x)$ is given by

$$\Psi_1(x) = \begin{cases} \exp(\mathrm{i}k_+^\mathrm{l} x)\begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_1 \exp(-\mathrm{i}k_+^\mathrm{l} x)\begin{pmatrix} 1 \\ 0 \end{pmatrix} + b_1 \exp(-\mathrm{i}k_-^\mathrm{l} x)\begin{pmatrix} 0 \\ 1 \end{pmatrix}, & x < 0 \\ c_1 \exp(\mathrm{i}k_+^\mathrm{r})\begin{pmatrix} \cos(\theta/2)\mathrm{e}^{-\mathrm{i}\phi/2} \\ \sin(\theta/2)\mathrm{e}^{\mathrm{i}\phi/2} \end{pmatrix} + d_1 \exp(\mathrm{i}k_-^\mathrm{r})\begin{pmatrix} -\sin(\theta/2)\mathrm{e}^{-\mathrm{i}\phi/2} \\ \cos(\theta/2)\mathrm{e}^{\mathrm{i}\phi/2} \end{pmatrix}, & x > L. \end{cases} \tag{2}$$

In this equation, $k_\pm^{l(r)} = \sqrt{2m(E - U_{l(r)} \mp h_{l(r)})/\hbar^2 - k_\parallel^2}$ is the spin-dependent wavevector along the x-direction in the left (right) FM electrode, E is the single-electron energy which is conserved when electrons tunnel through the junction, $\mathbf{k}_\parallel$ is the wavevector parallel to the interface between the different regions and is assumed to be conserved in the quantum tunnelling process and the explicit wavefunction $\mathrm{e}^{(\mathrm{i}k_\parallel y)}$ in every term is omitted in the above equation. Nevertheless, this is not a required condition for the following derivation. Ψ_1 describes a minority-spin electron coming from the left FM lead and being scattered in

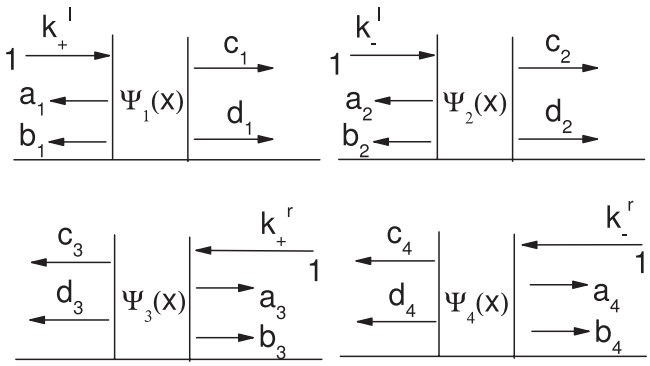

Figure 2. A schematic picture of four elementary scattering events. The injecting wavefunction of an electron will be scattered into four outgoing wavefunctions, two reflections and two transmissions; e.g., $\Psi_1(x)$ denotes a minority spin electron k^{l}_{+} in the left FM injecting into the middle region with two reflection (a_1 and b_1) and two transmission (c_1 and d_1) wavefunctions.

the middle region; the scattering coefficients a_1, b_1, c_1, and d_1 correspond to the normal reflection, Andreev-type reflection [8], transmission without branch crossing, and transmission with branch crossing [27], respectively. Here the branch crossing means the injected wavefunction with wavevector k^{l}_{+} will become the transmitted k^{r}_{-} or reflected k^{l}_{-} one due to the noncollinearity. For the other three scattering processes, Ψ_i as well as their coefficients a_i, b_i, c_i, and d_i as shown in figure 2 have the same meaning. It is noted that we have omitted the parallel plane wave component $\mathrm{e}^{\mathrm{i}\mathbf{k}_{\parallel}\vec{r}(y,z)}$ in the above equation. When we use the continuity of the derivatives of wavefunctions at the interfaces to determine these coefficients (a_i, b_i, c_i, and d_i), the parallel momentum $\mathbf{k}_{\parallel}$ should be explicitly taken into account.

With four elementary scattering wavefunctions like those in equation (2) above, the single-electron Green's function of the junction system can be worked out by the McMillan formula [23], which has been further developed by Kashiwaya and Tanaka [26]. This formula has been used to treat the Josephson current in a superconductor junction and relate directly the supercurrent to the Andreev reflection amplitudes. The Green's function $G^{\mathrm{r}}(x, x')$ is proportional to the direct product of the left-going wavefunctions (processes $i = 3, 4$ in figure 2) and the right-going wavefunctions (processes $i = 1, 2$), $G^{\mathrm{r}} \sim \Psi_{\mathrm{L}}(x)\hat{\Psi}_{\mathrm{R}}(x')$ for $x \leqslant x'$ and $G^{\mathrm{r}} \sim \Psi_{\mathrm{R}}(x) * \hat{\Psi}_{\mathrm{L}}(x')$ for $x \geqslant x'$, where the hat '^' denotes the conjugate process to the elementary scattering one shown in figure 2, and these conjugate scattering wavefunctions can be obtained by determining the Hermitian conjugate of only the spinor part of the wavefunction not including the spatial part of the wavefunction. The reflection and transmission coefficients $\tilde{a}_i$, $\tilde{b}_i$, $\tilde{c}_i$, and $\tilde{d}_i$ in four conjugate processes have the relations $\tilde{a}_i(\phi) = a_i(-\phi)$, $\tilde{b}_i(\phi) = b_i(-\phi)$, $\tilde{c}_i(\phi) = c_i(-\phi)$, and $\tilde{d}_i(\phi) = d_i(-\phi)$ ($i = 1 \ldots 4$), where ϕ is the azimuthal angle of the magnetization of the right FM. With these scattering wavefunctions of the elementary processes as well as their conjugate processes, the Green's function is then constructed in a linear combination as

$$G^{\mathrm{r}}(x, x', E) = \begin{cases} \alpha_1\Psi_3(x)\hat{\Psi}_1(x') + \alpha_2\Psi_3(x)\hat{\Psi}_2(x') + \alpha_3\Psi_4(x)\hat{\Psi}_1(x') \\ \quad + \alpha_4\Psi_4(x)\hat{\Psi}_2(x'), \qquad x \leqslant x' \\ \beta_1\Psi_1(x)\hat{\Psi}_3(x') + \beta_2\Psi_2(x)\hat{\Psi}_3(x') + \beta_3\Psi_1(x)\hat{\Psi}_4(x') \\ \quad + \beta_4\Psi_2(x)\hat{\Psi}_4(x'), \qquad x \geqslant x'. \end{cases} \tag{3}$$

Here $G^{\mathrm{r}}(x, x', E)$ is implicitly a function of the parallel momentum $k_{\parallel}$. The prefactors α_i and β_i $(i = 1 \ldots 4)$ can be determined by the boundary conditions that the Green's function fulfills,

$$\frac{\partial}{\partial x} G^{\mathrm{r}}(x, x', E)\bigg|_{x=x'+0} - \frac{\partial}{\partial x} G^{\mathrm{r}}(x, x', E)\bigg|_{x=x'-0} = \frac{2m_{\mathrm{e}}}{\hbar^2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{4}$$

$$G^{\mathrm{r}}(x, x', E)|_{x=x'+0} = G^{\mathrm{r}}(x, x', E)|_{x=x'-0}. \tag{5}$$

With these two equations, we can directly solve the prefactors α_i and β_i in equation (3), which are independent of the spatial position x. After some direct algebra, they read

$$\begin{aligned}
\alpha_1 &= \frac{z_+ c_4}{c_3 c_4 - d_3 d_4}, & \alpha_2 &= \frac{-z_- d_4}{c_3 c_4 - d_3 d_4}, \\
\alpha_3 &= \frac{-z_+ d_3}{c_3 c_4 - d_4 d_4}, & \alpha_4 &= \frac{z_- c_3}{c_3 c_4 - d_4 d_4}, \\
\beta_1 &= \frac{z_+ \tilde{c}_4}{\tilde{c}_3 \tilde{c}_4 - \tilde{d}_3 \tilde{d}_4}, & \beta_2 &= \frac{-z_- \tilde{d}_4}{\tilde{c}_3 \tilde{c}_4 - \tilde{d}_3 \tilde{d}_4}, \\
\beta_3 &= \frac{-z_+ \tilde{d}_3}{\tilde{c}_3 \tilde{c}_4 - \tilde{d}_3 \tilde{d}_4}, & \beta_4 &= \frac{z_- \tilde{c}_3}{\tilde{c}_3 \tilde{c}_4 - \tilde{d}_3 \tilde{d}_4},
\end{aligned} \tag{6}$$

where $z_{\pm} = \frac{m_{\mathrm{e}}}{\mathrm{i}\hbar^2 k^{\mathrm{l}}_{\pm}}$. In these solutions, we have employed some detailed balance conditions to facilitate our derivation, such as $a_i(\phi) = a_i(-\phi)$ $(i = 1 \ldots 4)$ and $k^{\mathrm{l}}_{-}\tilde{b}_1 = k^{\mathrm{l}}_{+} b_2$. Substituting these prefactors into equation (3), we obtained the Green's function in the left FM electrode $(x, x' < 0)$ as

$$\begin{aligned}
G^{\mathrm{r}}(x, x', E) &= \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}, \\
G_{11} &= z_+ \exp(\mathrm{i}k^{\mathrm{l}}_{+}|x - x'|) + a_1 z_+ \exp(-\mathrm{i}k^{\mathrm{l}}_{+} x - \mathrm{i}k^{\mathrm{l}}_{+} x'), \\
G_{12} &= z_- b_2 \exp(-\mathrm{i}k^{\mathrm{l}}_{+} x - \mathrm{i}k^{\mathrm{l}}_{-} x'), \\
G_{21} &= z_+ b_1 \exp(-\mathrm{i}k^{\mathrm{l}}_{+} x' - \mathrm{i}k^{\mathrm{l}}_{-} x), \\
G_{22} &= z_- \exp(\mathrm{i}k^{\mathrm{l}}_{-}|x - x'|) + a_2 z_- \exp(-\mathrm{i}k^{\mathrm{l}}_{-} x - \mathrm{i}k^{\mathrm{l}}_{-} x').
\end{aligned} \tag{7}$$

The Green's functions in other regions such as the middle region between the two FMs can also be constructed in a similar manner according to equation (3) with solutions given in equation (6). Although the Green's function is obtained here for the FM/FM junction, this method can also be applied to nonmagnetic junctions, and we believe the derivation procedure is universal in mesoscopic junction systems. When the retarded Green's function of the studied system is worked out, we can in principle calculate the physical observables that we want. Since we focus on an equilibrium tunnelling junction, the lesser Green's function is easily obtained by the formula [28]

$$G^{<}(x, x', E) = \left[G^{\mathrm{a}}(x, x', E) - G^{\mathrm{r}}(x, x', E)\right] f(E) \tag{8}$$

where $G^{\mathrm{a}}(x, x', E)$ is the advanced Green's function and $f(E)$ is the Fermi–Dirac distribution function. The lesser Green's function is related to the spectral function of the electron and is very useful for calculating some physical quantities as shown later.

3. ESC in the FM/FM junction

In this section, we first present a general formula of the spin current in an FM and then utilize the Green's function obtained in section 2 to work out the ESC in an FM/FM tunnelling junction.

Then, as an example, we calculated the ESC of a simple FM/FM junction with a delta function insulator barrier between the two FM electrodes.

The local spin density $\vec{s}(\mathbf{r}, t)$ at position $\mathbf{r}$ and time t is defined as

$$\vec{s}(\mathbf{r}, t) = \Psi^{\dagger}(\mathbf{r}, t)\hat{\vec{s}}\Psi(\mathbf{r}, t), \tag{9}$$

where $\Psi(\mathbf{r}, t)$ is a two-component wavefunction and $\hat{\vec{s}} = \frac{\hbar}{2}\hat{\vec{\sigma}}$ with $\hat{\vec{\sigma}}$ denoting the Pauli spin matrices. Taking the time derivative of $\vec{s}(\mathbf{r}, t)$ and using the Schrodinger equation for a single FM, the continuity equation of spin current density is given by

$$\frac{\partial \vec{s}}{\partial t} + \nabla \cdot \mathbf{J}_s + \mathbf{S} = 0, \tag{10}$$

where the respective spin current density $\mathbf{J}_s$ and the source term $\mathbf{S}$ are

$$\begin{aligned} \mathbf{J}_s &= \frac{\hbar^2}{4m_{\mathrm{e}}\mathrm{i}}\left[\Psi^{\dagger}\boldsymbol{\sigma}\nabla\Psi - (\nabla\Psi)^{\dagger}\boldsymbol{\sigma}\Psi\right], \\ \mathbf{S} &= \Psi^{\dagger}(\boldsymbol{\sigma} \times \mathbf{h})\Psi, \end{aligned} \tag{11}$$

where $\mathbf{h}$ is the magnetization of the FM leads (for example, $\mathbf{h}_{\mathrm{l}}$, $\mathbf{h}_{\mathrm{r}}$). $\mathbf{J}_s$ is the usual definition of spin current density and $\mathbf{S}$ is referred to as the spin source term or spin torque that leads to spin rotation when the spin is not collinear with the magnetization $\mathbf{h}$. Therefore, this spin source term gives rise to a spin current flow which is not included in $\mathbf{J}_s$. Using the lesser Green's function in equation (8), $G^{<}(xt, x't') = \mathrm{i}\langle\Psi^{\dagger}(x't')\Psi(xt)\rangle$ where $\langle\cdots\rangle$ is the quantum statistical average, the spin current density $\mathbf{J}_s$ and the source term $\mathbf{S}$ above in a steady state can be rewritten as [29]

$$\mathbf{J}_s = \frac{-\hbar^2}{4m_{\mathrm{e}}} \lim_{x'\to x}\left\{\frac{\partial}{\partial x'} - \frac{\partial}{\partial x}\right\}\int \frac{\mathrm{d}E}{2\pi}\mathrm{Tr}\left[\boldsymbol{\sigma}G^{<}(x', x)\right], \tag{12}$$

$$\mathbf{S} = \lim_{x'\to x}\int \frac{\mathrm{d}E}{2\pi}\mathrm{Tr}\left[(\mathbf{h} \times \boldsymbol{\sigma})G^{<}(x', x)\right], \tag{13}$$

where the trace is taken over the spin space. Substituting the Green's function given by equation (7) and equation (8) into the above equation, we obtain the ESC expression in the FM/FM junction as

$$J_s^x = \int \frac{\mathrm{d}E}{4\pi}\sum_{k_\parallel}\mathrm{Re}\left\{(k_+^1 - k_-^1)\left(\frac{b_2}{k_-^1} - \frac{b_1}{k_+^1}\right)\exp[-\mathrm{i}(k_+^1 + k_-^1)x]\right\} f(E), \tag{14a}$$

$$J_s^y = -\int \frac{\mathrm{d}E}{4\pi}\sum_{k_\parallel}\mathrm{Im}\left\{(k_+^1 - k_-^1)\left(\frac{b_2}{k_-^1} + \frac{b_1}{k_+^1}\right)\exp[-\mathrm{i}(k_+^1 + k_-^1)x]\right\} f(E), \tag{14b}$$

$$J_s^z = 0. \tag{14c}$$

Here the summation over $k_\parallel$ includes the contribution of all transverse modes to the spin current, where $k_\parallel$ is assumed to be conserved in the quantum tunnelling process. According to the above equation, the spin current density is determined by the Andreev-type reflection coefficients b_1 and b_2; this result is very similar to the formula of the Josephson current, which is directly related to the Andreev reflection coefficients [25, 26]. The spin current density in equation (14) is exponentially x dependent so that the integral over energy and transverse density of states will make it disappear in bulk FM not far away from the interface. This phenomenon is the same as the spin transfer effect in that the transmitted spin current is also dependent on position and will vanish after summing all possible transverse modes with different wavevectors. Stiles and Zagwill [6] estimated that the characteristic length of this spatial spin precession was about $1/k_{\mathrm{f}}$ (k_{f} is the Fermi wavevector) because only the electrons near the Fermi energy contribute to the electric current. For our case in equation (14), all the

band electrons contribute to the ESC so that $\mathbf{J}_s$ is expected to decay much more quickly with distance from the interface.

Apart from the spin current density $\mathbf{J}_s$, which can be calculated directly from its definition, additional spin current flow arises from the source term $\mathbf{S}$ according to equation (10). According to the Gaussian theorem, another spin current $\mathbf{J}_{\rm soc}$ can be defined from this source term as

$$J_{\rm soc}^{(x)} = \int_x^0 S^x \,{\rm d}x = \int \frac{{\rm d}E}{4\pi} \sum_{k_\parallel} {\rm Re}\left\{(k_+^1 - k_-^1)\left(\frac{b_2}{k_-^1} - \frac{b_1}{k_+^1}\right)(1 - \exp[-{\rm i}(k_+^1 + k_-^1)x])\right\} f(E) \tag{15a}$$

$$J_{\rm soc}^{y} = \int_x^0 S^y \,{\rm d}x = -\int \frac{{\rm d}E}{4\pi} \sum_{k_\parallel} {\rm Im}\left\{(k_+^1 - k_-^1)\left(\frac{b_2}{k_-^1} + \frac{b_1}{k_+^1}\right)(1 - \exp[-{\rm i}(k_+^1 + k_-^1)x])\right\} f(E). \tag{15b}$$

The z-component of $\mathbf{J}_{\rm soc}$ is zero. The x-dependent part of $\mathbf{J}_{\rm soc}$ is just the $-\mathbf{J}_s$ in equation (14), so that the sum of $\mathbf{J}_{\rm soc}$ and $\mathbf{J}_s$ is independent of x and keeps constant from the interface to the bulk FM, which does not disappear away from the interface,

$$J^x = \int \frac{{\rm d}E}{4\pi} \sum_{k_\parallel} {\rm Re}\left\{(k_+^1 - k_-^1)\left(\frac{b_2}{k_-^1} - \frac{b_1}{k_+^1}\right)\right\} f(E), \tag{16a}$$

$$J^y = -\int \frac{{\rm d}E}{4\pi} \sum_{k_\parallel} {\rm Im}\left\{(k_+^1 - k_-^1)\left(\frac{b_2}{k_-^1} + \frac{b_1}{k_+^1}\right)\right\} f(E). \tag{16b}$$

When a spin current $\mathbf{J}_s$ enters the bulk FM, it will be absorbed by the lattice in the FM because the magnetization $\mathbf{h}$ rotates the spin, and the sum of $\mathbf{J}_{\rm soc}$ and $\mathbf{J}_s$ therefore remains constant.

As an example, we calculated a simple case of the FM/FM tunnelling junction; the potential energy in both FMs is $U_{\rm l} = U_{\rm r} = 0$, and the insulator barrier is described by a delta barrier $U_0\delta(x)$, where U_0 denotes the strength of the barrier. The magnitudes of the magnetizations in the two FMs are equal, $h_{\rm l} = h_{\rm r}$. Using a simple quantum mechanics method, we obtain the Andreev-type reflection amplitudes as

$$\begin{aligned} b_1 &= 2(k_- - k_+)k_+ \sin\theta \exp({\rm i}\phi)/A, \\ b_2 &= 2(k_- - k_+)k_- \sin\theta \exp(-{\rm i}\phi)/A, \\ A &= k_-^2 + k_+^2 + 6k_+k_- - 2k_0^2 + 4{\rm i}k_0(k_+ + k_-) - (k_+ - k_-)^2\cos\theta, \end{aligned} \tag{17}$$

where $k_\pm = k_\pm^{l(r)}$ and $k_0 = mU_0/\hbar^2$. Substituting these quantities into equation (16), the total ESC is then given by

$$\mathbf{J} = \int \frac{{\rm d}E}{\pi} \sum_{k_\parallel} {\rm Im}[1/A(E)](k_+ - k_-)^2 f(E)\hat{\mathbf{h}}_{\rm l} \times \hat{\mathbf{h}}_{\rm r} \tag{18}$$

where $\hat{\mathbf{h}}_{\rm l}(0, 0, 1)$ and $\hat{\mathbf{h}}_{\rm r}(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ are just the unit vectors of the magnetizations in the left and right FMs. The ESC depends on the directions of the two magnetizations not only through the cross product term $\hat{\mathbf{h}}_{\rm l} \times \hat{\mathbf{h}}_{\rm r}$ but also on the quantity A in equation (17). In the finite barrier case, we can neglect the $\cos\theta$ term in A so that equation (18) can reproduce the result obtained with the linear response theory, i.e. the exchange coupling between two magnetizations leading to the ESC. In the opposite limit, zero barrier between two FMs, $U_0 = 0$, equation (18) indicates there is no ESC flowing in the junction. This is the pure quantum mechanics effect; the reflected spin direction by a barrier rotates with respect to the incident spin direction and an imaginary part can appear in the quantity A of equation (17), which was regarded [6] as one of the reasons that spin current is not conserved at the interface

when a polarized charge current flows through an NM/FM junction. For the ESC discussed here, the reflections by a barrier lead to an additional phase to the incident wavefunction, which is necessary for the formation of ESC in the noncollinear FM/FM junction. It is quite different from the Josephson effect between two superconductors, in which a Cooper pair carries supercurrent through the junction even with zero barrier.

4. Summary

We have studied the ESC flowing in the noncollinear FM/FM tunnelling junction by extending the McMillan Green's function method. The single-electron Green's function of the junction can be constructed by the scattering coefficients, reflection and transmission amplitudes, which can be directly calculated by a simple quantum mechanics method. In the derived formula of spin current density, we found that the exact result of the ESC is determined by the Andreev-type reflection amplitudes as in the Josephson effect, and at the strong barrier our result can reproduce the well known linear response. It was also found that when there is no barrier between two FMs the ESC will disappear,as the reflected spin state from a finite barrier has an additional phase, which is crucial for the formation of the ESC.

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